ASTON ACADEMY

Self-Study Unit for Further Mathematics

Summer 2020

Name:___________________________________

To be completed for September 2020

Introduction

This unit is designed to be completed by you over the summer holidays in preparation for starting the further mathematics course in September. The aim of this unit is to allow you an insight into what will be expected of you once the course begins. It is also a way of preparing you to be a good independent learner as it has been proven that the best A'Level students are also good independent learners.

To be a good independent learner you should be able to:

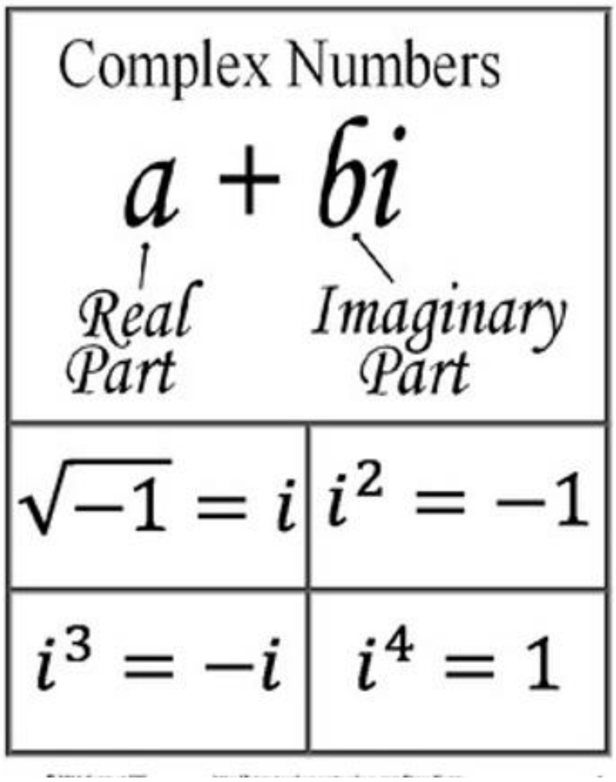
- Take responsibility for your own learning
- Be able to look information up for yourself
- Be able to structure questions to ask your teacher that will help you understand topics in more depth.

The idea of this unit is that it gives you an idea of some of the work that will be covered during the further maths course. It does include some examples to help you but you may also need to use other sources to help you such as the internet for example.

Good luck and we will expect this booklet to be handed to your core teacher in your first lesson with them in September.

C.Thompson

Section A – Complex Numbers



At GCSE you were told you could not square root a negative number, we can now do this but we need to introduce an imaginary number i. Above you can see that i = $\sqrt{-1}$

So $\sqrt{-36} = 6i$

We can also expand brackets containing an i and simplify our answers using the rules or i from above. Expand the double bracket as you would normally and then simplify.

(1 + i)(2 – i) = 2 + 2i – i – i^2

= 2 + i + 1 (as i^2 **= -1)**

= 3 + i

1. Simplify the following expressions:

a) $\sqrt{-49}$ = ______

b) $3 + \sqrt{-64}$ = ______

c) $\sqrt{-81} + \sqrt{-36}$ = ______

d) $3 + \sqrt{-64} + \sqrt{16}$ = ______

e) $\sqrt{-16} - \sqrt{-25}$ = ______

f) $5 + \sqrt{-4} + \sqrt{64}$ = ______

2. Expand and simplify the following expressions (Remember $i^2 = -1$) :

a) $(i - 4)(i + 3)$ = ______

______

b) $i(i + 8)$ = ______

______

c) $9i(i - 5)^2$ = ______

______

d) $(2i + 5) - 3i(2i + 6)$ = ______

______

e) $(5i + 4)(4i + 3)^2$ = ______

______

f) $(4i - 3) - 5i(3 + 6i)$ = ______

______

Section B – Matrices

Example

Given that $\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix}$ find

a **AB**

b **BA**.

Matrices with the same number of rows and columns are called **square** matrices.

The product of two square matrices of the same size will always give a square matrix of the same size.

a A is a 2X2 matrix and B is a 2X2 matrix so they can be multiplied and the product will be a 2X2 matrix.

This time there are four numbers to be found.

$$AB = \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$a = (-1) \times 4 + 0 \times 0 = -4$

$b = (-1) \times 1 + 0 \times (-2) = -1$

$c = 2 \times 4 + 3 \times 0 = 8$

$d = 2 \times 1 + 3 \times (-2) = -4$

a is the total of the first row multiplied by the first column.

b is the total of the first row multiplied by the second column.

c is the total of the second row multiplied by the first column.

d is the total of the second row multiplied by the second column.

So $AB = \begin{pmatrix} -4 & -1 \\ 8 & -4 \end{pmatrix}$

b BA will also be a 2 X 2 matrix

$$\begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ -4 & -6 \end{pmatrix}$$

First row times first column
$4 \times (-1) + 1 \times 2 = = -2$

First row times second column
$4 \times 0 + 1 \times 3 = 3$

Second row times second column
$0 \times 0 + (-2) \times 3 = -6$

Second row times first column
$0 \times (-1) + (-2) \times 2 = -4$

Inverse functions

- If $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $\mathbf{A}^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Your Turn

1. Multiply the following matrices together

   a) $\begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} . \begin{bmatrix} 6 & -6 \\ 3 & 0 \end{bmatrix}$

   b) $\begin{bmatrix} 6 \\ -3 \end{bmatrix} . \begin{bmatrix} -5 & 4 \end{bmatrix}$

   c) $\begin{bmatrix} -5 & -5 \\ -1 & 2 \end{bmatrix} . \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$

   d) $\begin{bmatrix} -3 & 5 \\ -2 & 1 \end{bmatrix} . \begin{bmatrix} 6 & -2 \\ 1 & -5 \end{bmatrix}$

2. Find the inverse of these matrices.

   a) $\begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$

   b) $\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$

   c) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

   d) $\begin{bmatrix} 2a & 3b \\ -a & -b \end{bmatrix}$

Section C – Series

Formula needed for this section:

$$\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$$

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{n^2}{4}(n+1)^2$$

Example

Evaluate **a** $\sum_{r=1}^{50} r$ **b** $\sum_{r=21}^{50} r$

a $\sum_{r=1}^{50} r = \frac{50 \times 51}{2} = 1275$

Substitute $n = 50$ in $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$

b $\sum_{r=21}^{50} r = \sum_{r=1}^{50} r - \sum_{r=1}^{20} r = 1275 - \frac{20 \times 21}{2}$

$= 1275 - 210$

$= 1065.$

Substitute $n = 20$ in $\frac{n}{2}(n+1)$

Calculate the sum of the series for each of the following.

1. $\sum_{1}^{20} r$ = ______________________________

2. $\sum_{1}^{15} r^2$ = ______________________________

3. $\sum_{1}^{5} r^3$ = ______________________________

4. $\sum_{1}^{25} r^2 + r$ =

5. $\sum_{1}^{10} r^3 + r$ =

6. $\sum_{1}^{10} r^3 - r$ =

Exam Style Questions

1. Show, using the formulae for $\sum r$ and $\sum r^2$, that

$$\sum_{r=1}^{n}(6r^2 + 4r - 1) = n(n + 2)(2n + 1).$$

(5)

2. Given that $\mathbf{X} = \begin{pmatrix} 2 & a \\ -1 & -1 \end{pmatrix}$, where a is a constant, and $a \neq 2$,

find $\mathbf{X}^{-1}$ in terms of a. ($\mathbf{X}^{-1}$ means the inverse)

(3)

3. $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$

Find **AB**.

(3)

4. $z = 5 - 3i$,

Express z^2 in the form $a + bi$, where a and b are real constants

(2)

Total 13

Conclusion

Questions I would like to ask:

Topics I am confident about: